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# Fayet-Iliopoulos terms in 5D orbifold supergravity

**Hiroyuki Abe, Kiwoon Choi and Ian-Woo Kim**

*Department of Physics, Korea Advanced Institute of Science and Technology*
*Daejeon 305-701, Korea*
*E-mail:* abe@hep.kaist.ac.kr, kchoi@hep.kaist.ac.kr, iwkim@hep.kaist.ac.kr

Abstract: We discuss a locally supersymmetric formulation for the boundary Fayet-Iliopoulos (FI) terms in 5-dimensional U(1) gauge theory on $S^1/Z_2$, using the four-form multiplier mechanism to introduce the necessary $Z_2$-odd FI coefficient. Some physical consequences of the boundary FI terms, e.g., supersymmetry/gauge symmetry breakings and the generation of 5D kink mass for hypermultiplet, are studied within the full supergravity framework. For models giving a flat spacetime geometry, the only meaningful deformation of vacuum configuration induced by the FI terms is a kink-type vacuum expectation value of the vector multiplet scalar field. On the other hand, for models giving an warped geometry, the boundary FI terms can lead to more interesting vacuum deformation breaking gauge symmetry and/or $N = 1$ supersymmetry, and also provide a non-trivial connection between the orbifold radius and the hyperscalar dynamics on the boundaries which may be useful for the radion stabilization.

Keywords: Field Theories in Higher Dimensions, Supersymmetry Breaking, Supergravity Models.

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## Contents



## 1. Introduction

Theories with extra dimension can provide an attractive mechanism to generate hierarchical structures in 4-dimensional (4D) physics such as the weak to Planck scale hierarchy [1, 2] and the hierarchical Yukawa couplings of quarks and leptons [3], as well as providing a new mechanism to break grand unified symmetry [4] and/or supersymmetry [5, 6, 7]. In regard to generating the scale and/or Yukawa hierarchies, quasi-localization of gravity [2, 8] and/or matter zero modes [3] in extra dimension is particularly interesting since it can generate exponentially different 4D scales and/or Yukawa couplings even when the fundamental parameters of the higher dimensional theory have similar magnitudes. A simple theoretical framework to implement the idea of quasi-localization would be 5D orbifold field theory on $S^1/Z_2$. For instance, in 5D orbifold supergravity (SUGRA), the Randall-Sundrum fine tuning [2] of the bulk and brane cosmological constants which is necessary for the gravity quasi-localization can be naturally obtained by gauging the $\mathrm{U}(1)_R$ symmetry with a $Z_2$-odd gauge coupling [9, 10, 11]. In the hypermultiplet compensator formulation of 5D off-shell SUGRA [10], this is equivalent to making the compensator hypermultiplet to have a nonzero $Z_2$-odd gauge coupling to the graviphoton. Also a nonzero 5D kink mass of matter hypermultiplet causing the quasi-localization of matter zero mode can be obtained by making the hypermultiplet to have a similar $Z_2$-odd gauge coupling [12, 13].

It has been noted that globally supersymmetric 5D U(1) gauge theory allows Fayet-Iliopoulos (FI) terms localized at the orbifold fixed points [14, 15, 16]. In globally supersymmetric 4D theories, FI term is allowed for generic U(1), and can lead to supersymmetry (SUSY) and/or gauge symmetry breakings. However extending the 4D global SUSY to SUGRA severely limits the possible FI terms. In 4D SUGRA, FI term is allowed only when the associated U(1) is either an $R$-symmetry [17] or a pseudo-anomalous U(1) endowed with the Green-Schwarz anomaly cancellation mechanism [18]. On the other hand, in 5D orbifold SUGRA, there can be a boundary FI term [15] proportional to

$$\frac{1}{2}\partial_y \epsilon(y) = \delta(y) - \delta(y - \pi R)\,,$$

even when U(1) is neither an $R$-symmetry nor a pseudo-anomalous symmetry, where $\epsilon(y)$ is the periodic sign function on the orbifold $S^1/Z_2$ whose fundamental domain is given by $0 \leq y \leq \pi R$. Such boundary FI terms generate a kink-type vacuum expectation value (VEV) of the scalar component of U(1) vector multiplet, thereby giving a kink mass to U(1)-charged matter hypermultiplets [15, 16, 19, 20] causing the quasi-localization of matter zero modes. It was also pointed out that such a boundary FI term can be induced at one-loop level even when it is absent at tree level [14, 15, 16, 19].

In order to have a boundary FI term proportional to $\partial_y \epsilon(y)$ in 5D orbifold SUGRA, one needs to introduce a $Z_2$-odd coupling $\xi_{FI}\epsilon(y)$. It is in fact non-trivial to introduce a $Z_2$-odd coupling in 5D orbifold SUGRA in a manner consistent with local supersymmetry. One known way is the mechanism of ref. [21] in which the $Z_2$-odd factor $\epsilon(y)$ appears as a consequence of the equations of motion of the four-form multiplier field. This procedure does not interfere with local SUSY, thus providing an elegant way to construct an orbifold SUGRA with $Z_2$-odd couplings, starting from a theory only with $Z_2$-even couplings.

In this paper we wish to discuss a locally supersymmetric formulation for the boundary FI terms in 5D $\mathrm{U}(1)_X$ gauge theory on $S^1/Z_2$. We apply the above mentioned four-form mechanism to the known formulation of 5D off-shell SUGRA [10]. We consider a class of simple models with boundary FI terms, and analyze the ground state solutions to examine possible symmetry breaking and also the generation of the hypermultiplet kink mass by the FI terms. In section 2, we derive the SUGRA action of bosonic fields containing a boundary FI term proportional to $\partial_y \epsilon(y)$, starting from the 5D off-shell SUGRA with four-form multiplier. We also discuss briefly the gauge anomalies involving the graviphoton which can have a kink-type fluctuation on the boundary in the presence of the FI coupling $\xi_{FI}\epsilon(y)$.

In section 3, we derive the Killing spinor conditions and energy functional in orbifold SUGRA models with FI terms for generic 4D Poincare-invariant background geometry. We then examine in section 4 the vacuum deformation caused by the boundary FI terms and the resulting physical consequences. We find that for models giving a flat spacetime geometry, the only meaningful deformation of vacuum configuration is a kink-type VEV of the vector multiplet scalar field. On the other hand, for models with warped geometry, the boundary FI terms can lead to more interesting vacuum deformations breaking gauge symmetries and/or the $N = 1$ SUSY. This FI-induced symmetry breaking in models with warped geometry is quite similar to the symmetry breaking in 4D SUGRA with gauged $\mathrm{U}(1)_R$ [17]. Indeed, if the parameters of the 5D model are adjusted to allow a 4D effective theory description of the FI-induced vacuum deformation, the low energy U(1)-gauge symmetry at scales below the Kaluza-Klein (KK) threshold scale becomes a $R$-symmetry with an effective 4D FI coefficient of $\mathcal{O}(k\xi_{FI}M_{Pl}^2)$ where $k$ is the AdS curvature of 5D geometry and $M_{Pl}$ is the 4D Planck mass. This is essentially a consequence of the kinetic mixing of $A^X_\mu$ with the graviphoton which corresponds to a $Z_2$-odd $\mathrm{U}(1)_R$ gauge boson in models with warped geometry. We finally note in section 4 that the boundary FI terms can provide also a non-trivial connection between the orbifold radius and the U(1)-charged hyperscalar dynamics on the boundaries, which may be useful for the radion stabilization.

## 2. 5D orbifold supergravity with boundary FI terms

In this section, we construct the action of 5D orbifold SUGRA containing the boundary FI terms for a bulk U(1) gauge symmetry which is originally neither an $R$-symmetry nor a pseudo-anomalous symmetry. We will use the off-shell formulation of 5D SUGRA on $S^1/Z_2$ which has been developed by Fujita, Kugo and Ohashi [10]. In this formulation, gravitational sector of the model is given by the Weyl multiplet and the central charge $\mathrm{U}(1)_Z$ vector multiplet $\mathcal{V}_Z$ which contain the fünfbein $e^m_\mu$ and the $Z_2$-odd graviphoton $A^Z_\mu$, respectively, and a consistent off-shell formulation is obtained by introducing a compensator hypermultiplet $\mathcal{H}_c$.[1] To discuss FI terms, we consider a minimal model with an ordinary $\mathrm{U}(1)_X$ vector multiplet $\mathcal{V}_X$ containing $Z_2$-even U(1) gauge field $A^X_\mu$, and also include a physical hypermultiplet $\mathcal{H}_p$. As it is a theory on $S^1/Z_2$, all physical and nonphysical 5D fields have the following boundary conditions:

$$\Sigma(-y) = Z\Sigma(y)\,, \qquad \Sigma(y+2\pi R) = \Sigma(y)\,,$$

where $Z^2 = 1$. It will be straightforward to extend our analysis to models containing arbitrary number of vector multiplets and hypermultiplets with more general boundary conditions.

In order to introduce the $Z_2$-odd coupling $\xi_{FI}\epsilon(y)$ for boundary FI terms, we apply the four-form mechanism of ref. [21] to the 5D off-shell SUGRA [10]. For this purpose, we introduce additional vector multiplet $\mathcal{V}_S$ as in ref. [10], so our model contains three U(1) vector multiplets at the starting point:

$$\begin{aligned}
\mathcal{V}_Z &= \left(\alpha,\, A^Z_\mu,\, \Omega^{Zi},\, Y^{Zij}\right),\\
\mathcal{V}_X &= \left(\beta,\, A^X_\mu,\, \Omega^{Xi},\, Y^{Xij}\right),\\
\mathcal{V}_S &= \left(\gamma,\, A^S_\mu,\, \Omega^{Si},\, Y^{Sij}\right),
\end{aligned}$$

where $M^A = (\alpha,\beta,\gamma)$ $(A = Z, X, S)$ are real scalar components, $\Omega^i_A$ $(i = 1, 2)$ are $\mathrm{SU}(2)_U$-doublet symplectic Majorana spinors, and $Y^{Aij}$ are $\mathrm{SU}(2)_U$-triplet auxiliary components. Note that $\alpha$ is $Z_2$-even for $Z_2$-odd $A^Z_{\underline{\mu}}$, while $\beta$ and $\gamma$ are $Z_2$-odd for $Z_2$-even $A^X_{\underline{\mu}}, A^S_{\underline{\mu}}$, and the $Z_2$ transformation of $\mathrm{SU}(2)_U$-doublet is given by $(\sigma_3)^i_{\ j}$. Throughout this paper, $\mu = (\underline{\mu}, y)$ represents the 5D coordinate directions with $\underline{\mu}$ representing the non-compact 4D coordinate directions, and $m = 0, 1, 2, 3, 4$ represents the 5D tangent space directions.

[1] The theories with a single compensator hypermultiplet can describe only the quaternionic hyperscalar manifolds $\mathrm{USp}(2, 2n_H)/\mathrm{USp}(2) \times \mathrm{USp}(2n_H)$ where $n_H$ is the number of physical hypermultiplets. To describe other types of hyperscalar geometry, one needs to introduce additional compensator hypermultiplets. However the physics of FI terms are mostly independent of the detailed geometry of the hyperscalar manifold. We thus limit the discussion to the theories with a single compensator hypermultiplet.

As for the hypermultiplets, we have

$$\begin{aligned} \text{compensator}: \quad & \mathcal{H}_c = (\mathcal{A}_i^x, \eta^x, \mathcal{F}_i^x)\,, \\ \text{physical}: \quad & \mathcal{H}_p = (\Phi_i^x, \zeta^x, F_i^x)\,, \end{aligned}$$

where $\mathcal{A}_i^x, \Phi_i^x$ $(x = 1, 2)$ are quaternionic hyperscalars, $\eta^x, \zeta^x$ are symplectic Majorana hyperinos, and $\mathcal{F}_i^x, F_i^x$ are auxiliary components. In the following, we will use frequently a matrix notation for hyperscalars, e.g.,

$$\Phi \equiv \begin{pmatrix} \Phi^{x=1}_{i=1} & \Phi^{x=1}_{i=2} \\ \Phi^{x=2}_{i=1} & \Phi^{x=2}_{i=2} \end{pmatrix} = \begin{pmatrix} \Phi_+ & \Phi_- \\ -\Phi_-^* & \Phi_+^* \end{pmatrix},$$

where $\Phi_\pm$ are $Z_2$ parity eigenstates. In this matrix notation, the symplectic reality condition and the $Z_2$ boundary condition are given by

$$\Phi^*(y) = i\sigma_2 \Phi(y) i\sigma_2^T\,, \qquad \Phi(-y) = \sigma_3 \Phi(y) \sigma_3\,. \tag{2.1}$$

An off-shell formulation for the four-form mechanism to generate $Z_2$-odd couplings in 5D orbifold SUGRA has been developed in ref. [10]. In this formulation, the four-form multiplier $H_{\mu\nu\rho\sigma}$ corresponds to the dual of the auxiliary scalar component of a linear multiplet $L_H$. This dualization is defined under the background of the Weyl multiplet and a vector multiplet $\mathcal{V}_B$, and leads to a three-form field $E_{\mu\nu\rho}$ as the dual of the constrained vector component of $L_H$. Here we choose $\mathcal{V}_B$ to be the central charge vector multiplet $\mathcal{V}_Z$, and then the superconformal invariant couplings of $L_H$ and $\mathcal{V}_S$ include

$$\mathcal{L}^{\text{bulk}}_{\text{4-form}} = e(Y^{Sij} - GY^{Zij})L_{ij} - \frac{1}{4!}\epsilon^{\lambda\mu\nu\rho\sigma}\left[\{F_{\lambda\mu}(A^S) - GF_{\lambda\mu}(A^Z)\}E_{\nu\rho\sigma} + \frac{1}{2}G\partial_\lambda H_{\mu\nu\rho\sigma}\right], \tag{2.2}$$

where $G = M^S/M^Z = \gamma/\alpha$, $L_{ij}$ is the $\mathrm{SU}(2)_U$ triplet scalar component of the linear multiplet $L_H$, $F_{\mu\nu}(A)$ is the field strength of the U(1) gauge field $A_\mu$, and $e = (-\det(g_{\mu\nu}))^{1/2}$. Once we have the above bulk interactions, $Z_2$-odd coupling constants can be obtained by introducing the following superconformal invariant boundary lagrangian density:

$$\mathcal{L}^{\text{brane}}_{\text{4-form}} = (\Lambda_0\delta(y) + \Lambda_\pi\delta(y - \pi R))\left[\frac{1}{4!}\epsilon^{\mu\nu\rho\sigma y}H_{\mu\nu\rho\sigma} + e_{(4)}\alpha(i\sigma_3)^{ij}L_{ij}\right],$$

where $e_{(4)} = (-\det(g_{\underline{\mu\nu}}))^{1/2}$ for the induced 4D metric $g_{\underline{\mu\nu}}$ on the boundaries. A detailed derivation of the above 4-form lagrangian densities can be found in ref. [10].

In the above lagrangian densities, $L_{ij}, E_{\mu\nu\rho}$ and $H_{\mu\nu\rho\sigma}$ play the role of lagrangian multipliers. By varying $H_{\mu\nu\rho\sigma}$, we obtain

$$\partial_\mu G = -2\delta^y_\mu(\Lambda_0\delta(y) + \Lambda_\pi\delta(y - \pi R))\,,$$

whose integrability condition leads to

$$\Lambda_0 = -\Lambda_\pi\,.$$

Taking the normalization $\Lambda_\pi = 1$, one finds

$$G = \gamma/\alpha = \epsilon(y)\,, \tag{2.3}$$

where $\epsilon(y) = y/|y|$ is the periodic sign-function obeying

$$\begin{aligned}\epsilon(y) &= -\epsilon(-y) = \epsilon(y + 2\pi R) = 1 \qquad (0 < y < \pi R)\,,\\ \partial_y\epsilon(y) &= 2(\delta(y) - \delta(y - \pi R))\,.\end{aligned}$$

Then the equations of motion for $E_{\mu\nu\rho}$ and $L_{ij}$ give

$$\begin{aligned}F_{\mu\nu}(A^S) &= \epsilon(y)F_{\mu\nu}(A^Z)\,,\\ Y^{Sij} &= \epsilon(y)Y^{Zij} + e^{-1}e_{(4)}\alpha(i\sigma_3)^{ij}(\delta(y) - \delta(y - \pi R))\,.\end{aligned} \tag{2.4}$$

Now using the relations (2.3) and (2.4), the redundant vector multiplet $\mathcal{V}_S$ can be *replaced* by the central charge vector multiplet $\mathcal{V}_Z$ multiplied by the $Z_2$-odd factor $\epsilon(y)$. This four-form mechanism provides an elegant way to obtain a locally supersymmetric theory of $\mathcal{V}_I$ $(I = Z, X)$ involving $Z_2$-odd couplings, starting from a locally supersymmetric theory of $\mathcal{V}_A$ $(A = Z, X, S)$ and the four-form multiplier multiplet involving only $Z_2$-even couplings.

The action of vector multiplets $\mathcal{V}_A$ $(A = Z, X, S)$ is determined by the norm function $\mathcal{N}$ which is a homogeneous cubic polynomial of the scalar components $M^A = (\alpha, \beta, \gamma)$. As a minimal model incorporating the boundary FI terms, we consider

$$\mathcal{N} = C_{ABC}M^AM^BM^C = \alpha^3 - \frac{1}{2}\alpha\beta^2 + \frac{1}{2}\xi_{FI}\alpha\beta\gamma\,.$$

Note that this form of $\mathcal{N}$ does not include any $Z_2$-odd coupling, thus the results of ref. [10] can be straightforwardly applied for our $\mathcal{N}$. Then the auxiliary components $Y^{Aij}$ appear in the bulk lagrangian as

$$e^{-1}\mathcal{L}_Y = -\frac{1}{2}\mathcal{N}_{AB}Y^{Aij}Y^B_{ij} + Y^A_{ij}\mathcal{Y}^{ij}_A\,, \tag{2.5}$$

where

$$\begin{aligned}\mathcal{N}_{AB} &= \frac{\partial^2\mathcal{N}}{\partial M^A\partial M^B}\,,\\ \mathcal{Y}^{ij}_A &= 2\left(\mathcal{A}^\dagger T_A\mathcal{A} - \Phi^\dagger T_A\Phi\right)^{ij}\,,\end{aligned}$$

for the U(1) charge operators $T_A$.

In this paper, we limit the discussion to the case that $T_A$ commute with the orbifolding $Z_2$ transformation $\Phi \to \sigma_3\Phi\sigma_3$. Then under the condition that the model does not contain any $Z_2$-odd coupling before the four-form multiplet is integrated out, the most general form of the hyperscalar U(1) charges consistent with the symplectic reality condition (2.1) is given by

$$\begin{aligned}\left(T_Z, T_X, T_S\right)\Phi &= \left(0,\, q,\, c\right)i\sigma_3\Phi\,,\\ \left(T_Z, T_X, T_S\right)\mathcal{A} &= \left(0,\, \tilde{q},\, -\tfrac{3}{2}k\right)i\sigma_3\mathcal{A}\,,\end{aligned} \tag{2.6}$$

where $q, c, \tilde{q}, k$ are real constants. If $\mathrm{U}(1)_X$ is not an $R$-symmetry, which is the case that we are focusing here, the compensator hyperscalar $\mathcal{A}$ is neutral under $\mathrm{U}(1)_X$, i.e.,

$$\tilde{q} = 0\,.$$

When $\tilde{q} \neq 0$, so $\mathrm{U}(1)_X$ is an $R$-symmetry, there can be additional FI terms both in the bulk and boundaries [22]. As we will see, in case that $\mathcal{A}$ has a nonzero $\mathrm{U}(1)_S$ charge $\frac{3}{2}k$, the central charge $\mathrm{U}(1)_Z$ becomes an $R$-symmetry with a $Z_2$-odd gauge coupling $\frac{3}{2}k\epsilon(y)$ after the four-form multiplet is integrated out. Such model has a ground state geometry being a slice of $\mathrm{AdS}_5$ with the AdS curvature $k$ [9, 10], so corresponds to the supersymmetric version of the Randall-Sundrum model.

Our goal is to derive the action of physical fields by systematically integrating out all non-physical degrees of freedom. We already noted that the equations of motion of $H_{\mu\nu\rho\sigma}, E_{\mu\nu\rho}$ and $L_{ij}$ result in the relations (2.3) and (2.4). Using these relations, we find first of all

$$\mathcal{L}^{\rm bulk}_{\text{4-form}} + \mathcal{L}^{\rm brane}_{\text{4-form}} = 0\,,$$

and

$$e^{-1}\mathcal{L}_Y = -\frac{1}{2}\tilde{\mathcal{N}}_{IJ}Y^{Iij}Y^J_{ij} + Y^I_{ij}\tilde{\mathcal{Y}}^{ij}_I + e^{-1}e_{(4)}\alpha(i\sigma_3)_{ij}\mathcal{Y}^{ij}_S(\delta(y) - \delta(y-\pi R))\,,$$

where

$$\begin{aligned}
\tilde{\mathcal{N}}_{IJ} &= \frac{\partial^2\tilde{\mathcal{N}}}{\partial M^I \partial M^J}\,,\\
\tilde{\mathcal{Y}}^{ij}_I &= 2(\mathcal{A}^\dagger t_I \mathcal{A} - \Phi^\dagger t_I \Phi)^{ij} - \frac{1}{2}\xi_{FI}e^{-1}e_{(4)}(i\sigma_3)^{ij}(\alpha^2\delta^X_I + \alpha\beta\delta^Z_I)(\delta(y) - \delta(y-\pi R))\,,\\
\mathcal{Y}^{ij}_S &= -3ik(\mathcal{A}^\dagger\sigma_3\mathcal{A})^{ij} - 2ic(\Phi^\dagger\sigma_3\Phi)^{ij}\,,
\end{aligned} \tag{2.7}$$

for $M^I = (\alpha, \beta)$ $(I = Z, X)$ and

$$\tilde{\mathcal{N}} = \mathcal{N}|_{\gamma=\epsilon(y)\alpha} = \tilde{C}_{IJK}M^IM^JM^K = \alpha^3 - \frac{1}{2}\alpha\beta^2 + \frac{1}{2}\xi_{FI}\epsilon(y)\alpha^2\beta\,. \tag{2.8}$$

We remark that the $\xi_{FI}$-term in $\tilde{\mathcal{N}}$ is same as the one noted in ref. [15]. Here the new U(1) generators $t_I$ for hyperscalars are given by

$$\begin{aligned}
\Big(t_Z,\, t_X\Big)\Phi &= \Big(c\epsilon(y),\, q\Big)i\sigma_3\Phi\,,\\
\Big(t_Z,\, t_X\Big)\mathcal{A} &= \Big(-\frac{3}{2}k\epsilon(y),\, 0\Big)i\sigma_3\mathcal{A}\,,
\end{aligned} \tag{2.9}$$

and $\tilde{\mathcal{N}}^{IJ}$ is the inverse matrix of $\tilde{\mathcal{N}}_{IJ}$. Note that although $\mathcal{L}_Y$ was a bulk action in the original theory, it gives boundary terms after the four-form multiplet is integrated out:

$$e^{-1}\mathcal{L}_Y \quad\to\quad \mathrm{tr}\left[i\sigma_3\left(\xi_{FI}\frac{(\alpha^2Y^X + \alpha\beta Y^Z)}{2} - \alpha\mathcal{Y}_S\right)\right]e^{-1}e_{(4)}(\delta(y) - \delta(y-\pi R))\,.$$

There appear additional boundary terms arising from the bulk kinetic term of $M^A = (\alpha, \beta, \gamma)$ which is given by

$$e^{-1}\mathcal{L}_{\rm kin} = -\frac{1}{4}\mathcal{N}\frac{\partial^2 \ln\mathcal{N}}{\partial M^A \partial M^B}\nabla_m M^A \nabla^m M^B\,.$$

Using $\gamma = \epsilon(y)\alpha$, we find

$$e^{-1}\mathcal{L}_{\rm kin} = -\frac{1}{4}\tilde{\mathcal{N}}\frac{\partial^2 \ln \tilde{\mathcal{N}}}{\partial M^I \partial M^J}\nabla_m M^I \nabla^m M^J + \frac{1}{2}\xi_{FI}\alpha^2\partial_4\beta\, e^{-1}e_{(4)}\Big(\delta(y) - \delta(y-\pi R)\Big) + \Delta\mathcal{L}_{\rm brane}\,, \tag{2.10}$$

where $M^I = (\alpha, \beta)$ and

$$\begin{aligned} \Delta\mathcal{L}_{\rm brane} = {} & \frac{1}{4}\xi_{FI}e^{-1}e_{(4)}(\delta(y) - \delta(y-\pi R))\tilde{\mathcal{N}}^{-1} \times \\ & \times \Big[(2\alpha^3\beta^2 - \xi_{FI}\epsilon(y)\alpha^4\beta)\partial_4\beta - (4\alpha^4\beta + \xi_{FI}\epsilon(y)\alpha^3\beta^2)\partial_4\alpha - \\ & - \xi_{FI}\alpha^4\beta^2 e^{-1}e_{(4)}(\delta(y) - \delta(y-\pi R))\Big]\,. \end{aligned} \tag{2.11}$$

If the $Z_2$-odd $\beta$ vanishes on the boundaries as was assumed in [10, 21], all operators in $\Delta\mathcal{L}_{\rm brane}$ would vanish also. However, as we will see, in the presence of the boundary FI terms, $\beta$ develops a kink-type of fluctuation on the boundaries. In this situation, one can *not* simply assume that operators involving $\beta$ vanish on the boundaries. The values of $\Delta\mathcal{L}_{\rm brane}$ depend on how to regulate $\beta$ across the boundary. This would cause a UV sensitive ambiguity in the theory, and makes it difficult to find the correct on-shell SUSY transformation and Killing spinor conditions on the boundaries.

It is in fact a generic phenomenon in orbifold field theory that boundary operators can produce a UV sensitive singular behavior of bulk fields on the boundary. Normally the resulting subtleties correspond to higher order effects in the perturbative expansion in powers of dimensionful coupling constants which is equivalent to an expansion in powers of $1/\Lambda$ for the cutoff scale $\Lambda$. Then the low energy physics below $\Lambda$ can be described in a UV insensitive manner by limiting the analysis to an appropriate order in the expansion. In our case, all the boundary terms appear in connection with the dimensionful $Z_2$-odd coupling constants $\lambda = (\xi_{FI}, k, c)$. For instance, $\xi_{FI}\epsilon(y)$ leads to the integrable boundary FI terms, while the $Z_2$-odd gauge couplings $\frac{3}{2}k\epsilon(y)$ and $c\epsilon(y)$ give rise to the integrable boundary tensions and hyperscalar mass-squares. It turns out that at leading order in the perturbative expansion in $\lambda = (\xi_{FI}, k, c)$, those boundary terms generate the following kink-type fluctuations

$$\beta \sim \xi_{FI}\epsilon(y)\,, \qquad \partial_y K \sim k\epsilon(y)\,, \qquad \partial_y \Phi_+ \sim c\,\Phi_+(0)\epsilon(y)\,, \tag{2.12}$$

for the spacetime metric $g_{\underline{\mu\nu}} = e^{2K}\eta_{\underline{\mu\nu}}$. Then the precise value of boundary operators involving $\Omega_- = (\beta, \partial_y K, \partial_y \Phi_+)$ would depend on how to regulate the singular fixed point and the behavior of $\Omega_-$ across the boundary. As in other cases, we can avoid this problem of UV sensitivity by truncating the theory at an appropriate order in the expansion in powers of $\lambda = (\xi_{FI}, k, c)$.

To make this point more explicit, we parametrize $M^I = (\alpha, \beta)$ in terms of a physical scalar field $\phi$ under the gauge fixing condition

$$\tilde{\mathcal{N}} = 1$$

in the unit with the 5D Planck mass $M_5 = 1$:

$$\begin{aligned}
\alpha &= \frac{\cosh^{2/3}(\phi)}{(1+\xi_{FI}^2\epsilon^2(y)/8)^{1/3}} \\
&= 1 + \frac{1}{3}\Big(\phi^2 - \frac{1}{8}\xi_{FI}^2\epsilon^2(y)\Big) + \mathcal{O}(\phi^4)\,, \\
\beta &= \frac{\cosh^{2/3}(\phi)[(2+\xi_{FI}^2\epsilon^2(y)/4)^{1/2}\tanh(\phi) + \xi_{FI}\epsilon(y)/2]}{(1+\xi_{FI}^2\epsilon^2(y)/8)^{1/3}} \\
&= \frac{1}{2}\xi_{FI}\epsilon(y) + \sqrt{2}\phi + \mathcal{O}(\phi^3)\,.
\end{aligned}$$

Then the very special manifold spanned by $\phi$ has the metric

$$g_{\phi\phi}(\phi) = -\frac{1}{2}\frac{\partial^2 \ln\tilde{\mathcal{N}}}{\partial M^I \partial M^J}\frac{\partial M^I}{\partial\phi}\frac{\partial M^J}{\partial\phi} = \frac{1+2\cosh(2\phi)}{3\cosh^2(\phi)} = 1 + \frac{1}{3}\phi^2 + \mathcal{O}(\phi^4)\,.$$

Obviously, $\phi$ is $Z_2$-odd for $Z_2$-odd $\beta$. For $\xi_{FI}\neq 0$, $\phi$ has a kink-type fluctuation of $\mathcal{O}(\xi_{FI})$ near the boundaries. Then, simply counting the powers of $\xi_{FI}$ in the limit $\xi_{FI}\ll 1$, one easily finds $\alpha = \mathcal{O}(1)$, $\beta = \mathcal{O}(\xi_{FI})$, $\partial_y\beta = \mathcal{O}(\xi_{FI})$, and $\partial_y\alpha = \mathcal{O}(\xi_{FI}^2)$ near the boundaries. With this power counting, the first boundary term in (2.10) is of the order of $\xi_{FI}^2$, while

$$\Delta\mathcal{L}_{\rm brane} = \mathcal{O}(\xi_{FI}^4)\,.$$

In the next section, we will explicitly show that the energy functional including only the boundary operators up to $\mathcal{O}(\xi_{FI}^2)$ takes the standard Bogomolny-squared form. This implies that the analysis at $\mathcal{O}(\xi_{FI}^2)$ can be a consistent (approximate) framework to examine the effects of the boundary FI terms in orbifold SUGRA. If the operators of $\Delta\mathcal{L}_{\rm brane}$ are included, the Killing spinor conditions will receive corrections of $\mathcal{O}(\xi_{FI}^3)$. However, the values of $\Delta\mathcal{L}_{\rm brane}$ severely depend on the way of regulating the singular functions $\epsilon(y)$ and $\delta(y)$ across the boundaries. It is thus expected that one needs informations on the UV completion of orbifold SUGRA in order to make a complete analysis including $\Delta\mathcal{L}_{\rm brane}$.

Upon ignoring the higher dimensional boundary terms of $\Delta\mathcal{L}_{\rm brane}$, after integrating out all auxiliary fields *other than* $Y^I$ $(I = Z, X)$, we find the following lagrangian density of bosonic fields:

$$\begin{aligned}
e^{-1}\mathcal{L}_{\rm bulk} &= -\frac{1}{2}R - \frac{1}{4}\tilde{a}_{IJ}F^I_{\mu\nu}F^{\mu\nu J} + \frac{1}{2}\tilde{a}_{IJ}\nabla^m M^I\nabla_m M^J + \frac{1}{8}e^{-1}\tilde{C}_{IJK}\epsilon^{\lambda\mu\nu\rho\sigma}A^I_\lambda F^J_{\mu\nu}F^K_{\rho\sigma} + \\
&\quad + {\rm tr}\Big[|\nabla_m\Phi|^2 - |\nabla_m\mathcal{A}|^2 - |V_m|^2 - M^I M^J(\Phi^\dagger t_I^\dagger t_J\Phi - \mathcal{A}^\dagger t_I^\dagger t_J\mathcal{A})\Big] - \\
&\quad - \frac{1}{2}{\rm tr}\Big[\tilde{\mathcal{N}}_{IJ}Y^{I\dagger}Y^J - 4Y^{I\dagger}\Big(\mathcal{A}^\dagger t_I\mathcal{A} - \Phi^\dagger t_I\Phi\Big)\Big]\,, \\
e_{(4)}^{-1}\mathcal{L}_{\rm brane} &= \Bigg[\frac{1}{2}\xi_{FI}\alpha^2({\rm tr}[i\sigma_3 Y^X] + e^{-1}e_{(4)}\partial_y\beta) + \frac{1}{2}\xi_{FI}\alpha\beta\,{\rm tr}[i\sigma_3 Y^Z] - \\
&\quad - 2\alpha\left(3k + \frac{3}{2}k\,{\rm tr}\Big[\Phi^\dagger\Phi\Big] + c\,{\rm tr}\Big[\Phi^\dagger\sigma_3\Phi\sigma_3\Big]\right)\Bigg](\delta(y) - \delta(y-\pi R))\,. \qquad (2.13)
\end{aligned}$$

where

$$\tilde{a}_{IJ} = -\frac{1}{2}\frac{\partial^2\ln\tilde{\mathcal{N}}}{\partial M^I\partial M^J}\,,$$

$$V_m = \frac{1}{2}\left(\Phi^\dagger(\nabla_m\Phi) - (\nabla_m\Phi)^\dagger\Phi\right). \tag{2.14}$$

Here the $2\times 2$ matrix valued compensator hyperscalar field can be chosen as

$$\mathcal{A} = \mathbf{1}_2\sqrt{1+\frac{1}{2}\mathrm{tr}[\Phi^\dagger\Phi]}\,, \tag{2.15}$$

which corresponds to one of the gauge fixing conditions in the hypermultiplet compensator formulation of off-shell 5D SUGRA [10].

The above action indeed includes the boundary FI term proportional to

$$D_X = -(\mathrm{tr}[i\sigma_3 Y^X] + e^{-1}e_{(4)}\partial_y\beta)\,,$$

which can be identified as the $D$-component of the $N=1$ vector superfield originating from the 5D vector multiplet $\mathcal{V}_X$. Note that this boundary FI term accompanies a bulk mixing between $\mathcal{V}_X$ and $\mathcal{V}_Z$, e.g., the kinetic mixing of $A^X_\mu$ and $A^Z_\mu$ and the bilinear mixing of $Y^Z$ and $Y^X$. According to the action (2.13), the on-shell value of $Y^I$ is given by

$$Y^I = \tilde{\mathcal{N}}^{IJ}\tilde{\mathcal{Y}}_J\,, \tag{2.16}$$

where $\tilde{\mathcal{Y}}_I$ are defined in (2.7). Then after integrating out the auxiliary components $Y^I$, the full 5D scalar potential is given by

$$\begin{aligned} V_{5D} &= \mathrm{tr}\left[M^I M^J\{\Phi^\dagger t_I^\dagger t_J\Phi - \mathcal{A}^\dagger t_I^\dagger t_J\mathcal{A}\} - \frac{1}{2}\tilde{\mathcal{N}}^{IJ}\tilde{\mathcal{Y}}_I^\dagger\tilde{\mathcal{Y}}_J\right] + \\ &+2e^{-1}e_{(4)}\alpha\left(3k + \frac{3}{2}k\,\mathrm{tr}\left[\Phi^\dagger\Phi\right] + c\,\mathrm{tr}\left[\Phi^\dagger\sigma_3\Phi\sigma_3\right]\right)(\delta(y) - \delta(y-\pi R))\,. \end{aligned} \tag{2.17}$$

With the above results, one easily finds

$$V_{5D} = -6k^2 + 6ke^{-1}e_{(4)}(\delta(y) - \delta(y-\pi R)) + \cdots\,,$$

where the ellipsis stands for field-dependent terms. This shows that a nonzero $\mathrm{U}(1)_Z$ charge of the compensator hyperscalar, i.e., $\frac{3}{2}k\epsilon(y)\sigma_3$, gives rise to the correctly tuned bulk and boundary cosmological constants yielding a warped Randall-Sundrum geometry with AdS curvature $k$. After the compensator gauge fixing (2.15), this $\mathrm{U}(1)_Z$ charge corresponds to a $\mathrm{U}(1)_R$ gauge charge. As a result, the graviphoton $A^Z_\mu$ becomes a $\mathrm{U}(1)_R$ gauge field and its auxiliary component $Y^Z$ has a bulk FI term $\sim k\epsilon(y)Y^Z$ which leads to the negative bulk cosmological constant in $V_{5D}$.

Also from the on-shell expression of the $N=1$ auxiliary components, e.g., the order parameter $F$ in eq. (3.4), one can see that the hypermultiplet obtains the following 5D mass determined by the $\mathrm{U}(1)_I$ ($I=Z,X$) charges and the $\mathrm{U}(1)_X$ FI coefficient:

$$\begin{aligned} m^2_{\Phi_\pm} &= \left(c_w^2 \pm c_w - \frac{15}{4}\right)k^2\epsilon^2(y) + (3\mp 2c_w)k(\delta(y)-\delta(y-\pi R)) + \mathcal{O}(\lambda\langle\phi\rangle^3)\,, \\ m_\zeta &= c_w k\epsilon(y) + \mathcal{O}(\langle\phi\rangle^3)\,, \end{aligned} \tag{2.18}$$

where we consider only the hyperscalar mass from the $F$-term potential, $\lambda = (\xi_{FI}, k, c)$, and

$$c_w k\epsilon(y) \equiv -\left(c\epsilon(y) + \frac{1}{2}q\xi_{FI}\epsilon(y) + \sqrt{2}q\langle\phi\rangle\right).$$

Note that this hypermultiplet mass reproduces the result of ref. [12] when $\xi_{FI} = 0$.

Let us briefly discuss the gauge anomalies in 5D orbifold SUGRA presented in this section. It is well known that anomalies in 5D orbifold field theory are localized on the 4D boundaries [23, 15, 24]. The gauge anomalies involving *only* $Z_2$-even gauge fields have been discussed extensively in the literatures, thus will not be discussed here besides a simple remark on the CS term of $Z_2$-even $A^X_\mu$ [25], $\mathcal{L}_{CS} = C_X\epsilon(y)A^X \wedge dA^X \wedge dA^X$, which may be necessary for the anomaly cancellation. To achieve a CS term with $Z_2$-odd CS coefficient $C_X\epsilon(y)$, one needs to introduce a new four-form multiplier. In fact, this new four-form multiplier can produce the boundary FI terms also, however it can not accommodate the $Z_2$-odd $\mathrm{U}(1)_Z$ gauge coupling $k\epsilon(y)$ which is necessary to get warped $\mathrm{AdS}_5$ geometry. Since the physical consequences of the boundary FI terms are independent of the details of the anomaly cancellation mechanism, we will not address this point further, and use the four-form multiplier of (2.2) which can generate the $Z_2$-odd couplings $\xi_{FI}\epsilon(y)$ and $k\epsilon(y)$ simultaneously.

The gauge anomalies containing a $Z_2$-odd gauge field may be considered to be simply vanishing under the assumption that $Z_2$-odd gauge field vanishes on the boundary. However we already noticed that introducing a $Z_2$-odd coupling can cause kink-type boundary fluctuations of $Z_2$-odd fields, making the boundary operators involving $Z_2$-odd fields non-vanishing in general. This observation applies to the graviphoton $A^Z_\mu$ which can have a kink-type boundary fluctuation $F^Z_{\mu\nu} \sim \xi_{FI}\epsilon(y)F^X_{\mu\nu}$ which is induced for instance by the kinetic mixing: $\tilde{a}_{ZX}F^Z_{\mu\nu}F^{X\mu\nu} \sim \xi_{FI}\epsilon(y)F^Z_{\mu\nu}F^{X\mu\nu}$. Although there may exist other dynamical mechanism to nullify the anomalies involving $Z_2$-odd $A^Z_\mu$, a simple way to avoid the problem is to choose the gauge charges of $A^Z_\mu$ to cancel the anomaly coefficients. For the $\mathrm{U}(1)_{Z,X}$ gauging (2.9), once the compensator gauge fixing (2.15) is made, $A^Z_\mu$ couples to fermions as

$$\begin{aligned}
\nabla_\mu\psi^i_\nu &= \left(\partial_\mu + \frac{3}{2}k\epsilon(y)i\sigma_3 A^Z_\mu + \cdots\right)^i_j \psi^j_\nu\,,\\
\nabla_\mu\Omega^i &= \left(\partial_\mu + \frac{3}{2}k\epsilon(y)i\sigma_3 A^Z_\mu + \cdots\right)^i_j \Omega^j\,,\\
\nabla_\mu\Psi &= \left(\partial_\mu - ic\epsilon(y)A^Z_\mu + \cdots\right)\Psi\,,
\end{aligned}$$

where $\psi^i_\mu$ and $\Omega^i$ are symplectic Majorana gravitinos and gauginos, respectively, $\Psi = \zeta^{x=1} = C_5^\dagger\gamma^0\zeta^{*x=2}$ ($C_5$: 5D charge conjugation matrix) denotes the Dirac fermion in hypermultiplet, and the ellipses stand for other gauge couplings. For simplicity, here we consider the case that these fermions are all periodic under $y \to y + 2\pi R$, and obey the following $Z_2$ boundary conditions:

$$\psi^i_\mu(-y) = (\sigma_3)^i_j\gamma_5\eta^\nu_\mu\psi^j_\nu(y)\,, \qquad \Omega^i(-y) = (\sigma_3)^i_j\gamma_5\Omega^j(y)\,, \qquad \Psi(-y) = \gamma_5\Psi(y)\,,$$

where $\eta_\mu^\nu = \mathrm{diag}(1,1,1,1,-1)$. Then the resulting graviphoton anomaly can be written as

$$\int \delta\left(\frac{\partial_\mu A^{Z\mu}}{\epsilon(y)\partial_\mu\partial^\mu}\right)\frac{1}{16\pi^2}\kappa^{ab}[\delta(y)+\delta(y-\pi R)]dy\wedge\left(\frac{F^a}{\epsilon^a(y)}\right)\wedge\left(\frac{F^b}{\epsilon^b(y)}\right),$$

where $\delta(\partial_\mu A^{Z\mu}/\partial_\mu\partial^\mu)$ corresponds to the $\mathrm{U}(1)_Z$ gauge variation, $F^a = \frac{1}{2}F^a_{\mu\nu}dx^\mu\wedge dx^\nu$, $\epsilon^a(y)=1$ for $Z_2$-even $A^a_\mu$, while $\epsilon^a(y)=\epsilon(y)$ for $Z_2$-odd $A^a_\mu$. Note that $Z_2$-odd gauge field in the above anomaly counter term is divided by the kink factor $\epsilon(y)$ to take into account that it has a kink-type fluctuation on the boundary. Here $F^{a=0}_{\mu\nu} = R_{\mu\nu}$ is the Riemann curvature two form and thus $\kappa^{00}$ corresponds to the coefficients of $\mathrm{U}(1)_Z$-gravity-gravity anomaly, while $F^{a\neq 0}_{\mu\nu}$ stand for the field strengths of $A^{Z,X}_\mu$ (and also other Yang-Mills gauge fields in the theory), so $\kappa^{ab}$ $(a,b\neq 0)$ are the coefficients of $\mathrm{U}(1)_Z$-gauge-gauge anomalies. It is in fact straightforward to find $\kappa^{ab}$ using the known results on the anomalies in orbifold field theory [23], yielding

$$\begin{aligned}
\kappa^{00} &= \frac{1}{2}\left[-\frac{3}{2}k\Big(-21+\sum_\Omega 1\Big)+\sum_\Psi c_\Psi\right]\epsilon^2(y)\,,\\
\kappa^{aa} &= \frac{1}{2}\left[-\frac{3}{2}k\sum_\Omega \mathrm{Tr}[t_a^2(\Omega)]+\sum_\Psi c_\Psi\,\mathrm{Tr}[t_a^2(\Psi)]\right]\epsilon^2(y)\,,\qquad (a\neq 0,Z)\\
\kappa^{ZZ} &= \frac{1}{2}\left[\Big(-\frac{3}{2}k\Big)^3\Big(3+\sum_\Omega 1\Big)+\sum_\Psi c_\Psi^3\right]\epsilon^6(y)\,,\\
\kappa^{Za} &= \frac{1}{2}\left[\Big(-\frac{3}{2}k\Big)^2\sum_\Omega \mathrm{Tr}[t_a(\Omega)]+\sum_\Psi c_\Psi^2\,\mathrm{Tr}[t_a(\Psi)]\right]\epsilon^4(y)\,,
\end{aligned}$$

where $-21$ and $3$ in $\kappa^{00}$ and $\kappa^{ZZ}$ come from the gravitino, and $t_a$ $(a\neq 0,Z)$ denote the gauge generators including the $\mathrm{U}(1)_X$ charge. Note that the above graviphoton anomalies are non-vanishing in general as the regulated values of $\epsilon^{2n}(y)\delta(y)$ are nonvanishing. Then a simple way to avoid the graviphoton gauge anomalies is to choose the involved gauge charges to make $\kappa^{ab}=0$. With $A^Z_\mu = \mathcal{O}(\xi_{FI}\epsilon(y))$, the $\mathrm{U}(1)_Z$-gravity-gravity and $\mathrm{U}(1)_Z$-$G_a$-$G_a$ ($G_a = Z_2$-even gauge group) anomalies correspond to the operators of $\mathcal{O}(\lambda\xi_{FI})$ $(\lambda = k,c)$ which have been consistently kept in our analysis. Although the $\mathrm{U}(1)_Z^3$ and $\mathrm{U}(1)_Z$-$\mathrm{U}(1)_Z$-$G_a$ anomalies correspond to higher order operators of $\mathcal{O}(\lambda^3\xi^3_{FI})$ and $\mathcal{O}(\lambda^2\xi^2_{FI})$, respectively, we require their coefficients to be cancelled also since they can not be cancelled by the gauge variation of local operators.

## 3. Killing spinor conditions and energy functional

In this section, we derive the Killing spinor equations and the energy functional in 5D orbifold SUGRA model with boundary FI terms for generic 4D Poincare invariant metric configuration:

$$ds^2 = e^{2K(y)}\eta_{\underline{\mu\nu}}(x)dx^{\underline{\mu}}dx^{\underline{\nu}} - dy^2\,. \tag{3.1}$$

We will employ the relation $e_4^y = e^{-1}e_{(4)} = 1$ frequently in the following. Applying the local SUSY transformations of the gravitino $\psi_\mu^i$, the gauginos $\Omega_I^i$, and the compensator and physical hyperinos $\eta^x, \zeta^x$, we find [10]

$$
\begin{aligned}
\delta\psi_\mu^i &= \left(\partial_\mu - \frac{1}{4}\omega_\mu^{ab}\gamma_{ab}\right)\varepsilon^i - \frac{1}{2}\delta_\mu^y(\Phi^\dagger\partial_y\Phi - \partial_y\Phi^\dagger\Phi)^i{}_j\varepsilon^j - \frac{1}{3}\frac{\tilde{\mathcal{N}}_I}{\tilde{\mathcal{N}}}(Y^I)^i{}_j\gamma_\mu\varepsilon^j\,, \\
\delta\Omega^{iI} &= -\frac{i}{2}\gamma_5\partial_y M^I\varepsilon^i + (Y^I)^i{}_j\varepsilon^j - \frac{1}{3}M^I\frac{\tilde{\mathcal{N}}_J}{\tilde{\mathcal{N}}}(Y^J)^i{}_j\varepsilon^j, \\
\delta\eta^x &= \left(i\gamma_5\partial_y - \frac{1}{2}i\gamma_5(\Phi^\dagger\partial_y\Phi - \partial_y\Phi^\dagger\Phi)_{ij}\mathcal{A}^{xj} - M^I(t_I)^x_{x'}\mathcal{A}^{x'}_i\right)\varepsilon^i + \mathcal{A}^x{}_i\frac{\tilde{\mathcal{N}}_I}{\tilde{\mathcal{N}}}(Y^I)^i{}_j\varepsilon^j\,, \\
\delta\zeta^x &= \left(i\gamma_5\partial_y - \frac{1}{2}i\gamma_5(\Phi^\dagger\partial_y\Phi - \partial_y\Phi^\dagger\Phi)_{ij}\Phi^{xj} - M^I(t_I)^x_{x'}\Phi^{x'}_i\right)\varepsilon^i + \Phi^x{}_i\frac{\tilde{\mathcal{N}}_I}{\tilde{\mathcal{N}}}(Y^I)^i{}_j\varepsilon^j\,, \qquad (3.2)
\end{aligned}
$$

where $\tilde{\mathcal{N}}_I = \partial\tilde{\mathcal{N}}/\partial M^I$, $\mathcal{A}_i^x$ and $Y^I$ are given by (2.15) and (2.16), respectively, and we use the convention $i\gamma_5 = \gamma^{\mu=y}$. Here the local SUSY transformation spinor $\varepsilon^i$ obeys the $Z_2$-orbifolding condition $\varepsilon^i(-y) = \gamma_5(\sigma_3)^i{}_j\varepsilon^j(y)$. From the above local SUSY transformations, one can find that the Killing spinor conditions for 4D Poincare invariant spacetime are given by

$$
\begin{aligned}
\left(\partial_y - \frac{1}{2}(\Phi^\dagger\partial_y\Phi - \partial_y\Phi^\dagger\Phi) + \frac{1}{6}iM^I\tilde{\mathfrak{Y}}_I\gamma_5\right)\varepsilon &= 0\,, \\
\left(\partial_y K + \frac{1}{3}iM^I\tilde{\mathfrak{Y}}_I\gamma_5\right)\varepsilon &= 0\,, \\
\left(-\frac{1}{2}\partial_y M^I + ig_{\phi\phi}^{-1}\frac{\partial M^I}{\partial\phi}\frac{\partial M^J}{\partial\phi}\tilde{\mathfrak{Y}}_J\gamma_5\right)\varepsilon &= 0\,, \\
\left(\partial_y\Phi - \frac{1}{2}\Phi(\Phi^\dagger\partial_y\Phi - \partial_y\Phi^\dagger\Phi)^\dagger + iM^I t_I\Phi\gamma_5 - \frac{1}{2}iM^I\Phi\tilde{\mathfrak{Y}}_I\gamma_5\right)\varepsilon &= 0\,, \\
\left(\partial_y\mathcal{A} - \frac{1}{2}\mathcal{A}(\Phi^\dagger\partial_y\Phi - \partial_y\Phi^\dagger\Phi)^\dagger + iM^I t_I\mathcal{A}\gamma_5 - \frac{1}{2}iM^I\mathcal{A}\tilde{\mathfrak{Y}}_I\gamma_5\right)\varepsilon &= 0\,. \qquad (3.3)
\end{aligned}
$$

These Killing spinor conditions can be rewritten as

$$
\begin{aligned}
\partial_y\varepsilon_+ + \left(\frac{i}{6}M^I\tilde{\mathfrak{Y}}_I\sigma_3 - \frac{1}{2}(\Phi^\dagger\partial_y\Phi - \partial_y\Phi^\dagger\Phi)\right)\varepsilon_+ &= 0\,, \\
\kappa \equiv \partial_y K + \frac{i}{3}M^I\sigma_3\tilde{\mathfrak{Y}}_I &= 0\,, \\
G \equiv \partial_y\phi - ig_{\phi\phi}^{-1}\frac{\partial M^I}{\partial\phi}\sigma_3\tilde{\mathfrak{Y}}_I &= 0\,, \\
F \equiv \partial_y\Phi - \frac{1}{2}\Phi(\Phi^\dagger\partial_y\Phi - \partial_y\Phi^\dagger\Phi)^\dagger + iM^I t_I\Phi\sigma_3 - \frac{i}{2}\Phi M^I\tilde{\mathfrak{Y}}_I\sigma_3 &= 0\,, \\
\mathcal{F} \equiv \partial_y\mathcal{A} - \frac{1}{2}\mathcal{A}(\Phi^\dagger\partial_y\Phi - \partial_y\Phi^\dagger\Phi)^\dagger + iM^I t_I\mathcal{A}\sigma_3 - \frac{i}{2}\mathcal{A}M^I\tilde{\mathfrak{Y}}_I\sigma_3 &= 0\,, \qquad (3.4)
\end{aligned}
$$

where $\tilde{\mathfrak{Y}}_I$ are given by (2.7), $\varepsilon = (\varepsilon^{i=1}, \varepsilon^{i=2})$, and $\varepsilon_+ = \frac{1}{2}(1+\gamma_5\sigma_3)\varepsilon$. In the above, $\kappa = 0$ corresponds to the gravitino Killing condition, $G = 0$ is the gaugino Killing condition, $F = 0$ is the physical hyperino Killing condition, and $\mathcal{F} = 0$ comes from the compensator hyperino Killing condition. In fact, $\mathcal{F} = 0$ is not independent from other Killing conditions since the compensator hypermultiplet is not a physical degree of freedom. However, here we treat it separately for later convenience. When the above Killing conditions are all satisfied, the Killing spinor is given by

$$\varepsilon_+(y) = \exp\left[\frac{1}{2}(K(y) - K(0)) + \frac{1}{2}\int_0^y dz\,(\Phi^\dagger\partial_z\Phi - \partial_z\Phi^\dagger\Phi)\right]\varepsilon_+(0)\,.$$

The boundary FI terms affect the gaugino Killing condition $G = 0$ through $\tilde{\mathcal{Y}}_X\partial\beta/\partial\phi$, thus gives an effect of $\mathcal{O}(\xi_{FI})$ to the Killing condition. On the other hand, the other Killing conditions are affected through $\beta\tilde{\mathcal{Y}}_X$ which gives an effect of $\mathcal{O}(\xi_{FI}^2)$ for $\beta = \mathcal{O}(\xi_{FI})$.

In fact, the above Killing conditions are not sufficient conditions for $N = 1$ SUSY preserving 4D Poincare invariant vacuum. The solution should be a stationary point of the energy functional with a vanishing vacuum energy density. For the 4D Poincare invariant geometry (3.1), the energy functional resulting from the 5D action (2.13) is given by

$$E = \int dy\;\mathcal{E}\,,$$

where

$$\begin{aligned}\mathcal{E} = e^{4K}\Bigg[&4\partial_y^2K + 10(\partial_yK)^2 + \frac{1}{2}g_{\phi\phi}(\partial_y\phi)^2 + \\ &+\mathrm{tr}\Bigg\{\partial_y\Phi^\dagger\partial_y\Phi - \partial_y\mathcal{A}^\dagger\partial_y\mathcal{A} + \frac{1}{4}|\partial_y\Phi^\dagger\Phi - \Phi^\dagger\partial_y\Phi|^2 + \\ &\qquad + M^IM^J(\Phi^\dagger t_I^\dagger t_J\Phi - \mathcal{A}^\dagger t_I^\dagger t_J\mathcal{A}) - \frac{1}{2}\tilde{\mathcal{N}}^{IJ}\tilde{\mathcal{Y}}_I^\dagger\tilde{\mathcal{Y}}_J + \\ &\qquad + \left\{2\alpha\sigma_3\left(\frac{3}{2}k\,\mathcal{A}^\dagger\sigma_3\mathcal{A} + c\,\Phi^\dagger\sigma_3\Phi\right) - \frac{1}{4}\xi_{FI}\alpha^2\partial_y\beta\,\mathbf{1}_2\right\}(\delta(y) - \delta(y-\pi R))\Bigg\}\Bigg]\,,\end{aligned} \tag{3.5}$$

for $\mathcal{A} = (1 + \frac{1}{2}\,\mathrm{tr}(\Phi^\dagger\Phi))^{1/2}\mathbf{1}_2$. It is then straightforward to find that

$$\begin{aligned}\mathcal{E} = e^{4K}\Bigg[&\frac{1}{2}g_{\phi\phi}G^\dagger G - 6|\kappa|^2 + \mathrm{tr}\Bigg\{|F|^2 - |\mathcal{F}|^2 - \frac{1}{2}|\partial_y\Phi^\dagger\Phi - \Phi^\dagger\partial_y\Phi|^2 - \\ &\qquad - \frac{1}{2}iM^I(\Phi^\dagger\partial_y\Phi - \partial_y\Phi^\dagger\Phi)[\mathcal{A}^\dagger t_I\mathcal{A} - \Phi^\dagger t_I\Phi,\,\sigma_3]\Bigg\}\Bigg] + \\ &+\partial_y\left[\frac{i}{2}e^{4K}M^I\,\mathrm{tr}\,\sigma_3\Big(\mathcal{A}^\dagger t_I\mathcal{A} - \Phi^\dagger t_I\Phi) + \mathrm{h.c.} + 4e^{4K}\partial_yK\right]\,,\end{aligned}$$

so the energy functional can be rewritten as

$$\begin{aligned}E = \int dy\,e^{4K}\Bigg(&\frac{1}{2}g_{\phi\phi}G^\dagger G - 6|\kappa|^2 + \mathrm{tr}\{|F|^2 - |\mathcal{F}|^2 - \frac{1}{2}|\Phi^\dagger\partial_y\Phi - \partial_y\Phi^\dagger\Phi|^2\} - \\ &- \frac{1}{2}iM^I\,\mathrm{tr}\{(\Phi^\dagger\partial_y\Phi - \partial_y\Phi^\dagger\Phi)[\mathcal{A}^\dagger t_I\mathcal{A} - \Phi^\dagger t_I\Phi,\,\sigma_3]\}\Bigg)\,.\end{aligned}$$

To arrive at this Bogomolny form starting from the 5D action (2.13), we have truncated the boundary operators of $\mathcal{O}(\lambda\xi_{FI}^2)$ $(\lambda = (\xi_{FI}, k, c))$ which are UV sensitive as their values depend on the way of regulating the kink fluctuations (2.12).

The hyperscalar part of the above energy functional is not written in the more familiar on-shell form [26]. We confirmed that our energy functional can be rewritten in the standard on-shell form involving the metric of the quaternionic hyperscalar manifold $\mathrm{USp}(2,2)/\,\mathrm{USp}(2)\times\mathrm{USp}(2)$. It is in fact more convenient to use the above form of the hyperscalar energy functional rather than the conventional on-shell form to discuss supersymmetric solutions. For instance, it immediately shows that a field configuration satisfying the Killing conditions (3.4) and also the stationary conditions,

$$\begin{aligned} \Phi^\dagger\partial_y\Phi - \partial_y\Phi^\dagger\Phi &= 0\,, \\ [\sigma_3, \mathcal{A}^\dagger t_I\mathcal{A} - \Phi^\dagger t_I\Phi] &= 0\,, \end{aligned} \tag{3.6}$$

corresponds to a supersymmetric solution with vanishing vacuum energy. A simple solution of the above stationary conditions is

$$\Phi = \begin{pmatrix} v(y) & 0 \\ 0 & v(y) \end{pmatrix}\,, \tag{3.7}$$

where $v$ is a real VEV. For this form of $\Phi$, one easily finds

$$F = \mathbf{1}_2\, f(y)\,, \qquad \mathcal{F} = \mathbf{1}_2\,\frac{v(y)}{\sqrt{1+v^2(y)}}(f(y) + v^{-1}(y)\Delta(y))\,,$$

where

$$\begin{aligned} f(y) &= \partial_y v - v(1+v^2)\left[\left(\frac{3}{2}k + c\right)\epsilon(y)\alpha(\phi) + q\beta(\phi)\right] + v\Delta(y)\,, \\ \Delta(y) &= -\frac{1}{2}\xi_{FI}\alpha^2(\phi)\beta(\phi)\,(\delta(y) - \delta(y-\pi R))\,, \end{aligned}$$

for which the energy functional is given by

$$E = \int dy\, e^{4K}\left(\frac{1}{2}g_{\phi\phi}|G|^2 - 6|\kappa|^2 + \frac{2}{1+v^2}|f|^2\right)\,, \tag{3.8}$$

where again we have truncated the UV sensitive boundary operators of $\mathcal{O}(\lambda\xi_{FI}^2)$ ($\lambda = (\xi_{FI}, k, c)$).

## 4. Vacuum deformation induced by boundary FI terms

In this section, we study the deformation of vacuum solution induced by the boundary FI terms. For models giving a flat spacetime geometry, the vacuum deformation induced by FI terms in 5D orbifold SUGRA is essentially same as the vacuum deformation in the rigid SUSY case, i.e., a kink-type VEV of the vector multiplet scalar: $\langle\phi\rangle = \mathcal{O}(\xi_{FI})$. On the other hand, for models giving an warped spacetime geometry, the boundary FI terms can lead to more interesting vacuum deformation breaking either SUSY and/or gauge symmetries.

### 4.1 Flat model

Let us first consider a model with the $\mathrm{U}(1)_Z$ charge $\frac{3}{2}k=0$, which gives a flat vacuum geometry. To proceed, we assume that the hyperscalar VEV is given by (3.7). Then the stationary conditions (3.6) are automatically satisfied, and

$$\begin{aligned}\tilde{\mathfrak{Y}}_Z &= -i\Big(2c\epsilon(y)v^2+\frac{1}{2}\xi_{FI}\alpha\beta(\delta(y)-\delta(y-\pi R))\Big)\sigma_3\,,\\ \tilde{\mathfrak{Y}}_X &= -i\Big(2qv^2+\frac{1}{2}\xi_{FI}\alpha^2(\delta(y)-\delta(y-\pi R))\Big)\sigma_3\,.\end{aligned}$$

The Killing conditions (3.4) give

$$\begin{aligned}\varepsilon_+(y) &= e^{(K(y)-K(0))/2}\varepsilon_+(0)\,,\\ \partial_y K &= -\frac{2}{3}v^2\left[c\epsilon(y)\alpha(\phi)+q\beta(\phi)\right]+\frac{2}{3}\Delta(y)\,,\\ \partial_y v &= v(1+v^2)[\,c\epsilon(y)\alpha(\phi)+q\beta(\phi)]-v\Delta(y)\,,\end{aligned}\tag{4.1}$$

and also

$$\begin{aligned}\partial_y\phi =\ & \frac{2}{g_{\phi\phi}(\phi)}\Bigg[v^2\Bigg(c\epsilon(y)\frac{\partial\alpha(\phi)}{\partial\phi}+q\frac{\partial\beta(\phi)}{\partial\phi})+\\ &+\frac{1}{4}\xi_{FI}\Bigg\{\alpha^2(\phi)\frac{\partial\beta(\phi)}{\partial\phi}+\alpha(\phi)\beta(\phi)\frac{\partial\alpha(\phi)}{\partial\phi}\Bigg\}(\delta(y)-\delta(y-\pi R))\Bigg]\,.\end{aligned}\tag{4.2}$$

With $\beta=\mathcal{O}(\xi_{FI})$, the FI-induced boundary source $\Delta(y)$ in the gravitino Killing condition leads to a boundary fluctuation of $K$ which is of the order of $\xi_{FI}^2$. Upon ignoring this small fluctuation of $K$ on the boundary, the Killing conditions of (4.1) are satisfied if

$$\partial_y K=0\,,\qquad v=0\,,$$

for which the spacetime geometry is flat:

$$ds^2=\eta_{\underline{\mu}\underline{\nu}}dx^{\underline{\mu}}dx^{\underline{\nu}}-dy^2\,,$$

and $\varepsilon(y)$ is a constant spinor:

$$\varepsilon(y)=\varepsilon(0)\,.$$

The remaining gaugino Killing condition (4.2) becomes

$$\begin{aligned}\partial_y\phi &= \frac{\xi_{FI}}{2g_{\phi\phi}(\phi)}\left\{\alpha^2(\phi)\frac{\partial\beta(\phi)}{\partial\phi}+\alpha(\phi)\beta(\phi)\frac{\partial\alpha(\phi)}{\partial\phi}\right\}(\delta(y)-\delta(y-\pi R)\\ &= 2\xi_{FI}\,I(\phi,\xi_{FI}\epsilon(y))\,(\delta(y)-\delta(y-\pi R))\,,\end{aligned}\tag{4.3}$$

where

$$I(X,Y)=\frac{\cosh^2(X)}{\sqrt{8+Y^2}}\left(1+\frac{2Y\sinh(2X)}{\sqrt{8+Y^2}(1+2\cosh(2X))}\right).\tag{4.4}$$

This gaugino Killing condition (4.3) determines $\phi$ as

$$\phi(y)=\frac{\hat{\xi}_{FI}}{2\sqrt{2}}\epsilon(y)\,,\tag{4.5}$$

where $\hat{\xi}_{FI}$ is a constant given by

$$\hat{\xi}_{FI} = \xi_{FI} + \mathcal{O}(\xi_{FI}^3)\,.$$

Here the corrections of $\mathcal{O}(\xi_{FI}^3)$ depend on how to regulate $\epsilon(y)$ and $\delta(y)$. Note that the FI-induced VEV of $\phi$ has a simple kink form as in the rigid SUSY case.

We thus find that in 5D orbifold SUGRA models with flat geometry the only meaningful vacuum deformation induced by the boundary FI term is a kink-type vector multiplet scalar VEV of $\mathcal{O}(\xi_{FI})$ as it does in the rigid SUSY case. One interesting consequence of the boundary FI term is the 5D hypermultiplet mass which would lead to the quasi-localization of hypermultiplet zero modes. Using the expression (2.18) for the hypermultiplet mass in SUGRA and the above result of $\phi$, one easily finds

$$\begin{aligned} m_{\Phi_\pm}^2 &= c_f^2\epsilon^2(y) \mp 2c_f(\delta(y)-\delta(y-\pi R)) + \mathcal{O}(\lambda\langle\phi\rangle^3)\,, \\ m_\zeta &= c_f\epsilon(y) + \mathcal{O}(\langle\phi\rangle^3)\,, \end{aligned}$$

where $c_f \equiv -(c+q\xi_{FI})$. Note that the SUGRA FI term gives a bare kink mass[2] $q\xi_{FI}\epsilon(y)/2$ in addition to the kink mass due to $\phi$.

### 4.2 AdS$_5$ model

A more interesting vacuum deformation can occur in models with warped geometry. Some implications of the boundary FI terms in warped geometry have been examined in ref. [24] using the $N=1$ superspace formulation. However as we will show explicitly, the analysis of [24] is based on a superspace action which does not respect the 5D general covariance and also did not include the FI-induced mixing between the graviphoton and the U$(1)_X$ gauge boson, thereby yielding incorrect results which are quite different from ours. In this subsection, we first discuss the FI-induced vacuum deformation in AdS$_5$ models using the full SUGRA formulation developed in section 2, and later discuss the $N=1$ superspace formulation and the 4D effective theory interpretation in the rigid SUSY limit. This later discussion will make it clear where the discrepancies between our results and ref. [24] arise from.

In AdS$_5$ models, we have $k \neq 0$, and then

$$\begin{aligned} \tilde{\mathcal{Y}}_Z &= 2i\epsilon(y)\left[-\frac{3}{2}k\left(1+\frac{1}{2}\,\mathrm{tr}(\Phi^\dagger\Phi)\right)\sigma_3 - c\,\Phi^\dagger\sigma_3\Phi\right] - \frac{i}{2}\xi_{FI}\alpha\beta\sigma_3(\delta(y)-\delta(y-\pi R))\,, \\ \tilde{\mathcal{Y}}_X &= -2iq\Phi^\dagger\sigma_3\Phi - \frac{i}{2}\xi_{FI}\alpha^2\sigma_3(\delta(y)-\delta(y-\pi R))\,. \end{aligned}$$

Then for the hyperscalar VEV given by (3.7), the Killing conditions (3.4) give

$$\begin{aligned} \partial_y K &= -k\epsilon(y)\alpha(\phi) - \frac{2}{3}v^2\left[\left(\frac{3}{2}k+c\right)\epsilon(y)\alpha(\phi) + q\beta(\phi)\right] + \frac{2}{3}\Delta(y)\,, \\ \partial_y\phi &= \frac{1}{g_{\phi\phi}(\phi)}\left[3k\epsilon(y)\frac{\partial\alpha(\phi)}{\partial\phi} + 2v^2\left\{\left(\frac{3}{2}k+c\right)\epsilon(y)\frac{\partial\alpha(\phi)}{\partial\phi} + q\frac{\partial\beta(\phi)}{\partial\phi}\right\} + \right. \end{aligned}$$

[2]If the sign of $\langle\phi\rangle$ determined by the gaugino Killing condition were opposite, this bare kink mass would cancel the kink mass by $\langle\phi\rangle$, yielding the total kink mass of $\mathcal{O}(\xi_{FI}^3)$.

$$+\frac{1}{2}\xi_{FI}\left\{\alpha^2(\phi)\frac{\partial\beta(\phi)}{\partial\phi}+\alpha(\phi)\beta(\phi)\frac{\partial\alpha(\phi)}{\partial\phi}\right\}(\delta(y)-\delta(y-\pi R))\Bigg], \quad (4.6)$$

$$\partial_y v = v(1+v^2)\left[\left(\frac{3}{2}k+c\right)\epsilon(y)\alpha(\phi)+q\beta(\phi)\right]-v\Delta(y)\,.$$

When $\xi_{FI}=0$, a supersymmetric solution of the Killing conditions is given by

$$K=-k|y|\,,\qquad \phi=v=0\,,$$

yielding the $AdS_5$ geometry

$$ds^2=e^{-2k|y|}\eta_{\underline{\mu}\underline{\nu}}dx^{\underline{\mu}}dx^{\underline{\nu}}-dy^2\,, \tag{4.7}$$

with an undetermined value of the orbifold radius $R$. The corresponding Killing spinor is

$$\varepsilon_+(y)=e^{-k|y|/2}\varepsilon_+(0)\,.$$

So when $\xi_{FI}=0$, this model corresponds to the supersymmetric Randall-Sundrum model.

If a nonzero FI term is turned on, this vacuum solution is deformed in various ways. To see what can happen, let us first consider the case with $c=q=0$. In this case, the gaugino Killing equation becomes

$$\begin{aligned}\partial_y\phi &= \frac{1}{g_{\phi\phi}(\phi)}\Bigg[3k\epsilon(y)(1+v^2)\frac{\partial\alpha(\phi)}{\partial\phi}+\frac{1}{2}\xi_{FI}\left\{\alpha^2(\phi)\frac{\partial\beta(\phi)}{\partial\phi}+\alpha(\phi)\beta(\phi)\frac{\partial\alpha(\phi)}{\partial\phi}\right\}\times\\ &\qquad\times(\delta(y)-\delta(y-\pi R))\Bigg]\\ &= \frac{3}{2}k\epsilon(y)(1+v^2)J(\phi,\xi_{FI}\epsilon(y))+2\xi_{FI}I(\phi,\xi_{FI}\epsilon(y))(\delta(y)-\delta(y-\pi R))\,,\end{aligned}$$

where $I(X,Y)$ is given by (4.4) and

$$J(X,Y)=\frac{8}{(8+Y^2)^{1/3}}\frac{\cosh^{5/3}(X)\sinh(X)}{1+2\cosh(2X)}\,.$$

Together with the boundary conditions $\phi(-y)=-\phi(y)$ and $\phi(-y+\pi R)=-\phi(y+\pi R)$, this gaugino Killing condition determine the boundary values of $\phi(y)$ as

$$\begin{aligned}\phi(0_+)-\phi(0_-) &= 2\phi(0_+)=\frac{\hat{\xi}_{FI}}{\sqrt{2}}\,,\\ \phi(\pi R_-)-\phi(\pi R_+) &= 2\phi(\pi R_-)=\frac{\hat{\xi}_{FI}}{\sqrt{2}}\,.\end{aligned} \tag{4.8}$$

However at the same time, the gaugino Killing condition with $k\neq 0$ enforces $\phi(y)$ to be monotonically increasing or decreasing at $0<y<\pi R$ since $XJ(X,Y)>0$ for all $X\neq 0$ and $J(X=0,Y)=0$. Obviously such bulk behavior of $\phi$ can not be compatible with the boundary values (4.8). It is easy to see that this incompatibility between the $Z_2$ boundary conditions and the gaugino Killing condition is not a consequence of the specific ansatz (3.7) for the hyperscalar VEV, but is true for generic hyperscalar VEV as long as $q=c=0$.

We also note that this incompatibility can not be cured by the possible modification of the Killing conditions due to the boundary operators suppressed by more powers of $\xi_{FI}$ and/or $k$. This means that if there is no U(1)-charged matter fields, the gaugino Killing condition can not be satisfied, so the $N=1$ SUSY is broken by the FI term.

In fact, according to our discussion in section 2, the models with $k \neq 0$ but $c=q=0$ appear to suffer from the graviphoton gauge anomalies. In models with nonzero $c$ and/or $q$ which would be free from the graviphoton gauge anomalies, the above conclusion of $N=1$ SUSY breaking would not be valid anymore due to the additional contribution from the hyperscalar VEVs. However, still it implies that either $N=1$ SUSY or gauge symmetry should be broken since the $N=1$ SUSY is broken if all gauge charged matter scalar fields have vanishing VEVs. This FI-induced symmetry breaking is quite similar to the symmetry breaking in 4D SUGRA with gauged $\mathrm{U}(1)_R$ [17]. Indeed in warped 5D models with boundary FI terms, if the parameters are adjusted to the values which allow the 4D effective theory description of the FI-induced vacuum deformation, the low energy U(1)-gauge symmetry at scales below the Kaluza-Klein threshold scale becomes a $\mathrm{U}(1)_R$ symmetry with an effective 4D FI coefficient of $\mathcal{O}(k\xi_{FI}M_{Pl}^2)$ where $M_{Pl}^2 = M_5^3(1-e^{-2\pi kR})/k$ is the 4D Planck mass square. This is essentially a consequence of the kinetic mixing of $A_\mu^X$ with $A_\mu^Z$ which corresponds to a $Z_2$-odd $\mathrm{U}(1)_R$ gauge boson in models with warped geometry.

To discuss the above point in more detail, let us consider the limit $\xi_{FI} \ll 1$ in the unit with $M_5 = 1$. In this limit, the gaugino Killing condition in (4.6) can be written as

$$\partial_y \left[ e^{-2k|y|} \left( \phi - \frac{\xi_{FI}}{2\sqrt{2}} \epsilon(y) \right) \right] = \frac{k\xi_{FI}}{\sqrt{2}} e^{-2k|y|} + 2v^2 e^{-2k|y|} \left( \sqrt{2} q + \frac{2}{3} \phi \left( \frac{3}{2} k + c \right) \epsilon(y) \right), \quad (4.9)$$

where we have ignored the terms higher order in $\xi_{FI}$ with $\phi = \mathcal{O}(\xi_{FI})$. This Killing equation contains a non-integrable piece $k\xi_{FI}e^{-2k|y|}/\sqrt{2}$ in the right-hand side, thus can not be satisfied unless this non-integrable piece is cancelled by a nonzero hyperscalar VEV $v$. If $v$ can be freely adjusted to satisfy the above gaugino Killing condition, $N=1$ SUSY would be preserved, while some gauge symmetries are broken by nonzero $v$. However if the model contains a boundary superpotential of $\Phi_+$ giving a $F$-term hyperscalar potential which would influence $v$, generically the gaugino Killing condition can not be satisfied, and both $N=1$ SUSY and gauge symmetry are broken simultaneously. At any rate, the above Killing condition implies $\langle \phi \rangle \approx \frac{\xi_{FI}}{2\sqrt{2}} \epsilon(y)$ upon ignoring small corrections suppressed by more powers of $\lambda = (\xi_{FI}, k, c)$.

In the rigid SUSY limit ignoring higher order terms in $\xi_{FI}, k$ and $c$, the above Killing condition can be described in $N=1$ superspace formulation. To construct the 5D action in $N=1$ superspace, let us note that the vector field component $A_\mu^\phi$ which forms a 5D vector multiplet with $\phi$ is given by a (scalar field-dependent) combination of $A_\mu^I$ $(I = Z, X)$, while the other orthogonal combination corresponds to the on-shell graviphoton $B_\mu$ belonging to the 5D on-shell SUGRA multiplet:

$$A_\mu^\phi = \tilde{a}_{IJ} g^{\phi\phi} \frac{\partial M^I}{\partial \phi} A_\mu^J \,,$$

$$B_\mu = \frac{1}{3}\frac{\partial\tilde{\mathcal{N}}}{\partial M^I}A^I_\mu\,,$$

where $M^I = (\alpha,\beta)$ and we have chosen appropriate normalization factors for $A^\phi_\mu$ and $B_\mu$. Due to the property $\frac{\partial\tilde{\mathcal{N}}}{\partial M^I}\frac{\partial M^I}{\partial\phi} = 0$, $A^\phi_\mu$ and $B^\mu$ do not have a kinetic mixing. However, it is the field basis of $A^I_\mu$ ($I = Z, X$), not of the field-dependent combinations $A^\phi_\mu$ and $B_\mu$, for which the constant (up to the $Z_2$-odd factor $\epsilon(y)$) coupling constants such as $k\epsilon(y), \xi_{FI}\epsilon(y), c\epsilon(y)$ and $q$ can be introduced. In the limit $\xi_{FI} \ll 1$, upon ignoring the terms higher order in $\xi_{FI}$ with $\phi = \mathcal{O}(\xi_{FI})$, one easily finds

$$\begin{aligned} A^X_\mu &= \sqrt{2}A^\phi_\mu + \Big(\frac{1}{2}\xi_{FI}\epsilon(y) + \sqrt{2}\phi\Big)B_\mu\,, \\ A^Z_\mu &= B_\mu + \frac{2}{3}\phi A^\phi_\mu\,. \end{aligned} \tag{4.10}$$

Note that the gaugino Killing equation (4.9) gives the correct effective $\mathrm{U}(1)_\phi$ charge of the hyperscalar field $\Phi_+$, $q_{\rm eff} = \sqrt{2}q + (\frac{3}{2}k + c)\epsilon(y)\frac{2}{3}\phi$, which can be determined by noting that $\Phi_+$ couples to the combination $qA^X_\mu + (\frac{3}{2}k + c)\epsilon(y)A^Z_\mu$ after the compensator gauge fixing (2.15).

In the rigid SUSY limit, $B_\mu$ can be simply integrated out to construct the effective action of $\phi$ and $A^\phi_\mu$. Since the gravitino Killing condition in (4.6) implies that the deformation of $K$ induced by $\xi_{FI}$ and/or $v$ is essentially of $\mathcal{O}(\xi^2_{FI})$ or $\mathcal{O}(v^2)$, one can use the $\mathrm{AdS}_5$ metric (4.7) to construct the $N = 1$ superspace action of $\phi$ and $A^\phi_\mu$. To this end, let us define the $N = 1$ vector superfield $V$ containing $A^\phi_{\underline{\mu}}$ and also the $N = 1$ chiral superfield $\chi$ containing $\phi$ and $A^\phi_y$. Then in the $\mathrm{AdS}_5$ background (4.7), the superspace action of $V$ and $\chi$ can be written as [27]

$$\begin{aligned} \int d^5x\Bigg[\bigg\{\frac{1}{4}\int d^2\theta\, W^\alpha W_\alpha + \text{h.c.}\bigg\} + \int d^4\theta\, C^* e^{-2k|y| - \frac{1}{3}g_R V} C\,\times \\ \times\bigg\{-3 + \gamma_{FI}\epsilon(y)\Big(\partial_y V - \frac{\chi + \chi^*}{\sqrt{2}}\Big) + \Big(\partial_y V - \frac{\chi + \chi^*}{\sqrt{2}}\Big)^2\bigg\}\Bigg]\,, \end{aligned} \tag{4.11}$$

where we are using the notations of ref. [28] for the $N = 1$ superfields, and explicitly include the chiral compensator superfield $C$ whose scalar component is given by $(\mathcal{A}^{x=1}_{i=1})^{2/3}$ in order to accommodate the feature that $A^\phi_\mu$ has a $A^Z_\mu$ component, thus has $R$-gauge couplings in the warped case. Here $g_R$ is a constant of $\mathcal{O}(k\xi_{FI})$ which can be uniquely determined by the matching with the full SUGRA action, while the other constant $\gamma_{FI}$ of $\mathcal{O}(\xi_{FI})$ can be rotated away by the field redefinition $\chi \to \chi + \gamma_{FI}\epsilon(y)/2\sqrt{2}$ up to small corrections of $\mathcal{O}(\xi^2_{FI})$. Note that the above action is the most general action of $V$ and $\chi$ (up to quadratic terms of $V$) which is invariant under the superspace realization of $\mathrm{U}(1)_\phi$:

$$V \to V + \Lambda + \Lambda^*\,, \qquad \chi \to \chi + \sqrt{2}\partial_y\Lambda\,, \qquad C \to e^{\frac{1}{3}g_R\Lambda}C\,,$$

and also the global Weyl rescaling

$$C \to e^\sigma C\,, \qquad e^{-2k|y|} \to e^{-2\sigma}e^{-2k|y|} \qquad (\sigma = \text{constant})$$

which originates from the feature that the scalar component of $C$ corresponds to the conformal factor of the 4D metric, i.e., of $\eta_{\underline{\mu\nu}}$, which defines the $N=1$ superspace. Then the kinetic term matching determines the relative normalizations between $A^\phi_\mu, \phi$ and the $N=1$ superfields $V, \chi$ as

$$V = \theta\sigma^{\underline{\mu}}\bar{\theta}A^\phi_{\underline{\mu}}\,, \qquad \chi = \frac{1}{\sqrt{2}}(\phi + iA^\phi_y) + \eta\xi_{FI}\epsilon(y)\,,$$

where $\eta$ is an arbitrary constant of order unity.

After the superconformal gauge fixing $C=1$, the above action gives a term linear in $V$:

$$\int d^4\theta\, \tilde{\xi}_{FI}(y)\, V\,,$$

where

$$\tilde{\xi}_{FI}(y) = g_R e^{-2k|y|} - \partial_y\Big[e^{-2k|y|}(\gamma_{FI}\epsilon(y) - 2\sqrt{2}\mathrm{Re}(\chi))\Big]\,. \tag{4.12}$$

Comparing the $D$-flat condition from the superspace action (4.11) with the gaugino Killing condition (4.9), one easily finds

$$g_R = -\sqrt{2}k\xi_{FI}\,.$$

For this value of $g_R$, upon ignoring higher order corrections suppressed by more powers of $\lambda = (\epsilon_{FI}, k, c)$, the $N=1$ superspace action (4.11) reproduces correctly the 5D energy density (3.8) coming from nonzero $G$, and also the gravitino (gaugino) $\mathrm{U}(1)_R$ coupling to $A^\phi_\mu$, i.e., $\frac{3}{2}k\epsilon(y)\frac{2}{3}\phi \approx \frac{1}{4}g_R$, which comes from the $\mathrm{U}(1)_R$ coupling of $A^Z_\mu$.

At leading order in $\xi_{FI}$, the zero modes of $V$ and $C$ have a constant wavefunction. Then the 4D effective action of these zero modes is given by

$$\int d^4x\left[\left\{\frac{1}{4g_4^2}\int d^2\theta\, W^\alpha W_\alpha + \mathrm{h.c.}\right\} - 3M^2_{Pl}\int d^4\theta\, C^* e^{-\frac{1}{3}g_R V}C\right],$$

where the 4D gauge coupling $g_4^2 = 1/2\pi RM_5$ and the 4D Planck mass $M^2_{Pl} = M_5^3(1 - e^{-2\pi kR})/k$. This is obviously a 4D $N=1$ superspace action for a gauged $\mathrm{U}(1)_R$ symmetry with FI coefficient

$$\xi_{4D} = \oint dy\, \tilde{\xi}_{FI}(y) = -g_R M^2_{Pl}\,.$$

If $\sqrt{\xi_{4D}}$ is small enough to be well below the Kaluza-Klein threshold scale $M_{KK} \approx k/(e^{\pi kR} - 1)$, the vacuum deformation induced by boundary FI-terms in warped geometry can be described within the framework of 4D effective theory in which $\mathrm{U}(1)_\phi$ is a $R$-symmetry with FI coefficient $\xi_{4D} = -\sqrt{2}k\xi_{FI}M^2_{Pl}$.

Let us now compare our results with the results of [24] which have been produced the following integrable form of $\tilde{\xi}_{FI}$ in the original version:

$$\tilde{\xi}_{FI}(y) = \partial_y\Big[\epsilon(y)(A + Bke^{-k|y|})\Big]\,, \tag{4.13}$$

where $A$ and $B$ are some constants. The guidelines taken by [24] to obtain this form of $\tilde{\xi}_{FI}$ were (i) the integrability condition $\oint dy\, \tilde{\xi}_{FI}(y) = 0$ and (ii) the form of the power-law divergent one-loop FI counter terms. However as we have noted, if one introduces the $Z_2$-odd FI coupling $\xi_{FI}\epsilon(y)$ together with another $Z_2$-odd coupling $k\epsilon(y)$ for warped geometry in the framework of full 5D SUGRA, the low energy U(1) gauge symmetry below the KK threshold scale becomes inevitably a $R$-symmetry, thus the integrability condition (i) does not hold anymore. In fact, if one introduces a nonzero $\mathrm{U}(1)_X$-charge of the compensator $\mathcal{A}_i^x$, i.e., $\tilde{q}$ of eq. (2.6), which corresponds to the 5D $R$-charge for the $Z_2$-even $A_\mu^X$ [22], and tunes $\tilde{q}$ as $\tilde{q} = \frac{1}{4}\xi_{FI}k$ to cancel the FI-mixing-induced effective $R$-charge $-\sqrt{2}k\xi_{FI}$, one can make the resulting $g_R$ to vanish and $\tilde{\xi}_{FI}$ to satisfy the integrability condition (i). However obviously this is not a generic feature of the 5D SUGRA, but requires a fine tuning of parameters.

As for the point (ii), the original loop calculation of ref. [24] employs a constant cutoff scale $\Lambda_{\rm cut}$ in the 4D metric frame of $\eta_{\underline{\mu\nu}}$, not the correctly red-shifted position-dependent cutoff $e^{-k|y|}\Lambda_{\rm cut}$. This is equivalent to using the position-dependent cutoff $e^{k|y|}\Lambda_{\rm cut}$ for the scales measured by the original 5D metric $g_{\mu\nu}$ which is related to $\eta_{\underline{\mu\nu}}$ by the warp factor: $g_{\underline{\mu\nu}} = e^{-2k|y|}\eta_{\underline{\mu\nu}}$. Obviously, such cutoff violates 5D general covariance, thus not acceptable. Indeed, (4.13) does not respect the 5D general covariance. To see this, let us consider some terms of the $D$-component of $V$. The quadratic term $\int d^5x\, D^2$ in $\int d^2\theta\, W^\alpha W_\alpha$ must be originating from $\int d^5x\, e(Y^X)^2$ where $e = (-\det(g_{\mu\nu}))^{1/2} = e^{-4k|y|}$ and the auxiliary component $Y^X$ is a 5D general coordinate scalar. This uniquely fixes the field redefinition $D = e^{-2k|y|}Y^X$. Then the first term of (4.13) gives a 5D action

$$A\int d^5x \int d^4\theta\, \partial_y\epsilon(y)V = A\int d^5x\, e^{2k|y|}e_{(4)}[(\delta(y) - \delta(y-\pi R)]Y^X\,,$$

for the induced metric $e_{(4)} = (-\det(g_{\underline{\mu\nu}}))^{1/2} = e^{-4k|y|}$, which obviously does not respect the 5D general covariance.

So far, we have discussed the qualitative features of the vacuum deformation induced by the boundary FI-terms in models with warped geometry. To examine the vacuum deformation quantitatively, we analyze numerically the Killing equations to find a supersymmetric solution for some cases with nonzero $c$ and/or $q$. We find that in this case there exists an appropriate profile of $\langle\Phi_+\rangle = v(y)$ which allows all conditions for supersymmetric vacuum to be satisfied. So $N = 1$ SUSY is recovered, but at the expense of having a non-trivial hyperscalar VEV breaking the gauge symmetry. Also in some cases, there can be a sizable modification of the profile of $\phi$ and the warp factor exponent $K$. In figure 1, we present the resulting supersymmetric solutions for some parameter values of $k, c, q$ and $\xi_{FI}$. Note that $\phi$ can be significantly deformed from the simple kink shape, which may result in a significant change of the zero mode wavefunction of the hypermultiplet which couples to $\beta$. An interesting feature of this class of solutions is that the orbifold radius $R$ is determined if the hyperscalar VEV at one of the fixed points is fixed. In our model, still one combination of $v(0)$ (or $v(\pi R)$) and $R$ is undetermined by the equations of motion, so corresponds to a flat direction of the supersymmetric vacuum. However if one introduces a proper mechanism to fix $v$ at the fixed points, e.g., an appropriate boundary superpotential of $\Phi_+$, then the radion $R$ is accordingly stabilized. In this sense, the boundary FI term can provide a new way to stabilize the radion of warped geometry in 5D orbifold SUGRA models.

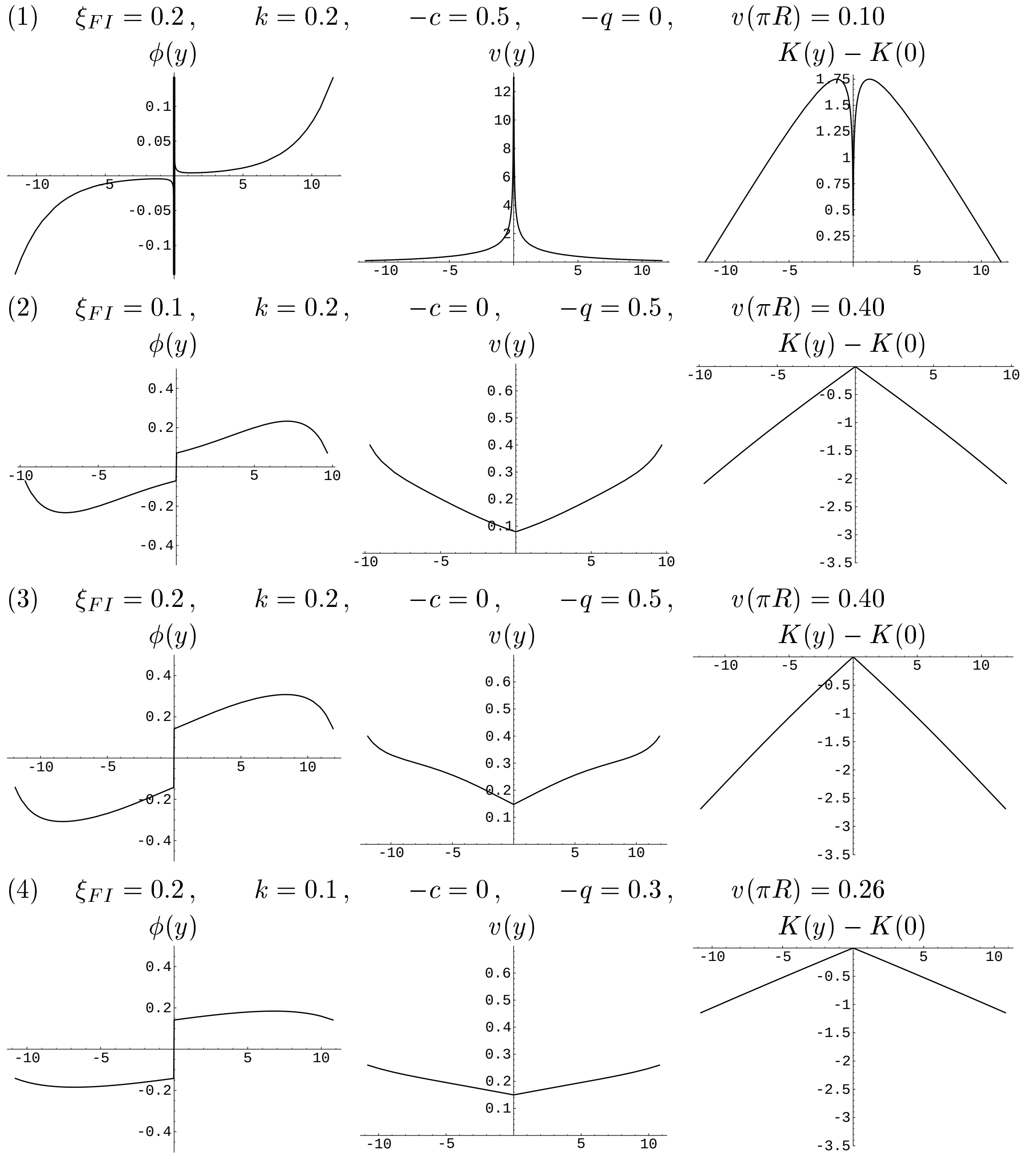


**Figure 1:** Supersymmetric vacuum configurations in models with $\xi_{FI} \neq 0$, $k \neq 0$ and $c, q \neq 0$. All curves end at $y = \pm\pi R$ in the horizontal axis. All parameters are given in the unit with the 5D Planck scale $M_5 = 1$. Note that the orbifold radius $R$ is determined if the value of $v(0)$ (or $v(\pi R)$) is fixed. Still a combination of $R$ and $v(0)$ is undetermined by the equations of motion.

## 5. Conclusion

In this paper, we presented a locally supersymmetric formulation for the boundary FI terms in 5D U(1) gauge theory on $S^1/Z_2$. We introduced a four-form multiplet to generate the $Z_2$-odd FI coefficient $\xi_{FI}\epsilon(y)$ within the 5D off-shell SUGRA on orbifold. The same four-form multiplier can be used to introduce the correct bulk and brane cosmological constants for the Randall-Sundrum warped geometry as well as the hypermultiplet kink masses for quasi-localized matter zero modes. We then examined the deformation of vacuum configuration triggered by the FI terms within the full SUGRA framework, and also the resulting physical consequences such as the supersymmetry/gauge symmetry breakings and the generation of 5D kink masses for hypermultiplets. It is found that for models giving a flat spacetime geometry, the only meaningful deformation of vacuum configuration is a kink-type vector multiplet scalar VEV of $\mathcal{O}(\xi_{FI})$ as in the rigid SUSY case. On the other hand, for models giving an warped $\mathrm{AdS}_5$ geometry, the boundary FI terms can lead to more interesting vacuum deformations, breaking gauge symmetry and/or $N=1$ supersymmetry. It is also noted that in the warped cases the boundary FI term can provide a non-trivial connection between the orbifold radius and the U(1)-charged hyperscalar dynamics on the boundaries, which may be useful for the radion stabilization.

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**Note added:** after this paper has been submitted, the authors of ref. [24] corrected the radiatively-induced FI terms to a form consistent with our results. According to the revised version of [24], when one takes the correct position dependent cutoff $\Lambda_{\rm cut}e^{-k|y|}$ in the metric frame of $\eta_{\underline{\mu\nu}}$, the power-divergent radiative corrections yield the 4D effective FI coefficient $\xi_{4D} = \oint dy\, \tilde{\xi}_{FI}(y) \propto \Lambda^2(1-e^{-2\pi kR}) \propto \Lambda^2 k M_{Pl}^2$ as suggested by our SUGRA analysis.

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